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I. N. Sadikov

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LAMINAR HEAT EXCHANGE IN THE INLET SECTION OF A
RECTANGULAR CHANNEL

I. N. Sadikov

ABSTRACT

Investigation of heat transfer in the inlet section of a rectangular channel with a laminar flow of incompressible fluid passing through the channel and a uniform or nonuniform temperature field arising at the inlet. Formulas are presented for the distribution of temperatures and Nusselt numbers along a side of a channel cross section.

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Author

It was shown in (Ref. 1) that, for the case of laminar flow, it is not permissible to transform the law of resistance, derived for a circular tube, to other transverse cross-sections by replacing the normal radius by the hydraulic radius. Thus, for example, the resistance coefficient for a circular tube is expressed by the formula

$$\zeta = 16 \operatorname{Re},$$

while for a rectangular tube, it is expressed by

$$\zeta = C \operatorname{Re},$$

* Note: Numbers in the margin indicate pagination in the original foreign text.

where $C = 14.225$ for a tube having a square cross-section, and $C = 24$ for a flat slit. This is also confirmed by data presented in the work (Ref. 2), which discusses the effect of the ratio between the side lengths of a rectangle upon the friction coefficient for the case of laminar flow. It is thus clear that for the case of laminar flow there is no basis for transforming data on the Nu number for circular tubes to rectangular tubes, by analogy with the transfer of heat and momentum.

It should be noted that laminar flow is observed in the inlet section of a tube for Re numbers which are larger than the critical number. Thus, for example, in (Ref. 2) and (Ref. 3), it was shown that for $Re > Re_{cr}$ there is a region of laminar flow to which the dip in the pressure curve corresponds in the inlet section of a rectangular channel. The presence of a laminar heat exchange region in the inlet section of a tube has been noted experimentally in several works, for example, in (Ref. 4, 5, 6), and the laminar flow existed up to $Re = 10^5$ numbers. Therefore, a theoretical study of laminar heat exchange in the inlet section of a rectangular channel is of great practical importance.

Since the temperature field at the entrance to a particular channel is frequently nonuniform in regenerators and compact heat exchangers, it is of considerable interest to examine the effect of temperature field nonuniformity at the entrance upon the heat exchange conditions in the inlet section of a channel and on the temperature distribution along its perimeter.

The energy equation of the boundary layer, for the stable flow of an incompressible liquid with constant physical properties, and in the absence of energy dissipation has the form (Ref. 7):

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right). \quad (1) \quad /424$$

Let us compare the convective terms $u \frac{\partial T}{\partial x}$, $v \frac{\partial T}{\partial y}$, $w \frac{\partial T}{\partial z}$ in this equation. Since, in the inlet section of a rectangular channel, the flow around each of its walls is similar to the flow over an infinite flat plate, in order to compare the convective terms included in the energy equation we can utilize the Blasius solution for the velocity distribution in the case of flow over a flat plate. This is also valid for the temperature field in the case of $Pr = 1$. It was shown in the study (Ref. 8) that the relationship $v \frac{\partial T}{\partial y} / u \frac{\partial T}{\partial x} \Big|_{Pr=1}$, in the section of the boundary layer where the longitudinal velocity component changes from $u = 0$ to $u = 0.95$, is close to a constant quantity, equaling 0.5. Only at a considerable distance from the wall, almost outside of the boundary layer limits, this relationship begins to change considerably.

On the basis of the statements given above, one can write

$$v \frac{\partial T}{\partial y} / u \frac{\partial T}{\partial x} = \text{const}, \quad w \frac{\partial T}{\partial z} / u \frac{\partial T}{\partial x} = \text{const}$$

and the energy equation for three-dimensional flow in the boundary layer assumes the form

$$\epsilon' u \frac{\partial T}{\partial x} = k \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right), \quad (2)$$

where

$$\epsilon' = 1 + v \frac{\partial T}{\partial y} / u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} / u \frac{\partial T}{\partial x}.$$

At the channel entrance, the magnitude of the velocity is constant over the cross-section and equals U . As was shown in the work (Ref. 9), if the longitudinal velocity component u is replaced by U , which is the average value over the cross-section, small changes are introduced in the solution of the equation, which can be taken into account in the correction ϵ , which is made in the equation. We then obtain

$$\epsilon U \frac{\partial T}{\partial x} = a \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right). \quad (3)$$

The magnitude of the correction ϵ was calculated in the work (Ref. 18):

$$\epsilon = 0.346 \text{Pr}^{-\frac{1}{3}}.$$

The equation which is obtained (3) is an equation of non-stationary thermal conductivity, and the methods to be employed in solving it are widely known (Ref. 10, 11).

Introducing the dimensionless variables $\xi = x/h$, $\eta = y/h$, $\zeta = z/h$ - where h is half of the distance between the walls of a rectangular channel which are perpendicular to the y -axis - and introducing the dimensionless temperature $\theta = (T - T_0)/T_0$ (T_0 is the characteristic temperature of the liquid at the entrance cross-section), we obtain

$$\text{--- } \epsilon \text{Pe} \frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial^2 \theta}{\partial \zeta^2}. \quad (4) \quad \underline{/425}$$

In the case of a given heat current through the walls of a rectangular channel, the boundary conditions are described in the following way:

$$\begin{aligned}
\zeta &= 0, \quad \theta = \beta_y \tau_1 + \gamma_y \tau_1^2 + \beta_z \zeta + \gamma_z \zeta^2, \\
\tau_1 &= \pm 1, \quad \frac{\partial \theta}{\partial \tau_1} = \pm K_{0y} \pm K_{1y} \zeta, \\
\zeta &= \pm \zeta_0, \quad \frac{\partial \theta}{\partial \zeta} = \pm K_{0z} \pm K_{1z} \zeta.
\end{aligned} \tag{5}$$

where $\zeta_0 = z_0/h$ is the distance from the x-axis to the walls in the direction of the z-axis.

The solution of equation (4) for the inlet and boundary conditions (5) can be represented as the sum of the solutions for the two following equations which give the temperature distribution in an infinite flat channel:

$$\begin{aligned}
\epsilon Pe \frac{\partial \theta_y}{\partial \zeta} &= \frac{\partial^2 \theta_y}{\partial \tau_1^2}, & \zeta &= 0, \quad \theta_y = \beta_y \tau_1 + \gamma_y \tau_1^2, \\
\tau_1 &= \pm 1, \quad \frac{\partial \theta_y}{\partial \tau_1} &= \pm K_{0y} \pm K_{1y} \zeta;
\end{aligned} \tag{6}$$

$$\begin{aligned}
\epsilon Pe \frac{\partial \theta_z}{\partial \zeta} &= \frac{\partial^2 \theta_z}{\partial \zeta^2}, & \zeta &= 0, \quad \theta_z = \beta_z \zeta + \gamma_z \zeta^2, \\
\zeta &= \pm \zeta_0, \quad \frac{\partial \theta_z}{\partial \zeta} &= \pm K_{0z} \pm K_{1z} \zeta.
\end{aligned} \tag{7}$$

It can be readily seen that the solution of equation (4), which satisfies the boundary conditions (5), is the sum of the solutions for equations (6) and (7):

$$\theta = \theta_y + \theta_z.$$

Solutions of equation (6) were obtained in the work (Ref. 12). If we apply these to the inlet section of the channel ($\zeta \gg 100$) for the number $Re \gg 3 \cdot 10^3$, we obtain

$$\begin{aligned}
\theta &= 2(K_{0y} - 2\gamma_y - \beta_y) \sqrt{\frac{\zeta}{\epsilon Pe}} \operatorname{ierfc} \left(\frac{1 - \tau_1}{2} \sqrt{\frac{\epsilon Pe}{\zeta}} \right) + \\
&+ 8K_{1y} \zeta \sqrt{\frac{\zeta}{\epsilon Pe}} \operatorname{i^3erfc} \left(\frac{1 - \tau_1}{2} \sqrt{\frac{\epsilon Pe}{\zeta}} \right) + \beta_y \tau_1 + \gamma_y \tau_1^2 + \\
&+ 2(K_{0z} - 2\gamma_z \zeta_0 - \beta_z) \sqrt{\frac{\zeta}{\epsilon Pe}} \operatorname{ierfc} \left(\frac{\zeta_0 - \zeta}{2} \sqrt{\frac{\epsilon Pe}{\zeta}} \right) + \\
&+ 8K_{1z} \zeta \sqrt{\frac{\zeta}{\epsilon Pe}} \operatorname{i^3erfc} \left(\frac{\zeta_0 - \zeta}{2} \sqrt{\frac{\epsilon Pe}{\zeta}} \right) + \beta_z \zeta + \gamma_z \zeta^2.
\end{aligned} \tag{8}$$

If we know the temperature distribution within the channel, we can find the mean temperature of the liquid and the wall temperature. We can use these to calculate the Nu number. We finally obtain

$$\begin{aligned} \text{Nu}|_{\zeta=\zeta_0} &= A_1/B_1, \quad A_1 = K_{0y} + K_{1y}\zeta, \\ B_1 &= 2(K_{0y} - 2\gamma_y - \beta_y) \sqrt{\zeta/\varepsilon \text{Pe}} \operatorname{ierfc}\left(\frac{1-\gamma_1}{2} \sqrt{\varepsilon \text{Pe} \zeta}\right) + \beta_y \gamma_1 + \gamma_y \gamma_1^2 + \\ &+ 8K_{1y}\zeta \sqrt{\varepsilon \text{Pe}} \operatorname{ierfc}\left(\frac{1-\gamma_1}{2} \sqrt{\varepsilon \text{Pe} \zeta}\right) + 1.1284(K_{0z} - 2\gamma_z \zeta_0 - \beta_z) \sqrt{\zeta/\varepsilon \text{Pe}} + \\ &+ 0.752K_{1z}\zeta \sqrt{\zeta/\varepsilon \text{Pe}} + \beta_z \zeta_0 + \gamma_z \zeta_0^2 - \left[\frac{K_{0z}\zeta + K_{1z}\zeta^2}{\zeta_0 \varepsilon \text{Pe}} + \frac{\gamma_z \zeta_0^2}{3} + \right. \\ &\quad \left. + \frac{K_{0y}\zeta + K_{1y}\zeta^2}{\varepsilon \text{Pe}} + \frac{\gamma_y}{3} \right]; \end{aligned} \quad (9) \quad /426$$

$$\begin{aligned} \text{Nu}|_{\zeta=\zeta_0} &= A_2/B_2, \quad A_2 = K_{0y} + K_{1y}\zeta, \\ B_2 &= 1.1284(K_{0y} - 2\gamma_y - \beta_y) \sqrt{\zeta/\varepsilon \text{Pe}} + \\ &+ 0.752K_{1y}\zeta \sqrt{\zeta/\varepsilon \text{Pe}} + \beta_y + \gamma_y + \\ &+ 2(K_{0z} - 2\gamma_z \zeta_0 - \beta_z) \sqrt{\zeta/\varepsilon \text{Pe}} \times \\ &\times \operatorname{ierfc}\left(\frac{\zeta_0 - \zeta}{2} \sqrt{\varepsilon \text{Pe} \zeta}\right) + \\ &+ 8K_{1z}\zeta \sqrt{\zeta/\varepsilon \text{Pe}} \operatorname{ierfc} \times \\ &\times \left(\frac{\zeta_0 - \zeta}{2} \sqrt{\varepsilon \text{Pe} \zeta}\right) + \gamma_z \zeta_0 + \gamma_z \zeta_0^2 - \\ &- \left[\frac{K_{0z}\zeta + K_{1z}\zeta^2}{\zeta_0 \varepsilon \text{Pe}} + \frac{\gamma_z \zeta_0^2}{3} + \right. \\ &\quad \left. + \frac{K_{0y}\zeta + K_{1y}\zeta^2}{\varepsilon \text{Pe}} + \frac{\gamma_y}{3} \right]. \end{aligned} \quad (10)$$

For a uniform temperature field at the entrance and a constant heat current along the perimeter and the length of the channel

($K_{0y} = K_{0z} = K_0$, $K_{1y} = K_{1z} = 0$), we obtain

$$\begin{aligned} \text{Nu}|_{\zeta=\zeta_0} &= \frac{1}{D}, \\ D &= 1.1284 \sqrt{\frac{\zeta}{\varepsilon \text{Pe}}} + 2 \sqrt{\frac{\zeta}{\varepsilon \text{Pe}}} \operatorname{ierfc}\left(\frac{\zeta_0 - \zeta}{2} \sqrt{\frac{\varepsilon \text{Pe}}{\zeta}}\right) - \frac{\zeta(1 + \zeta_0)}{\zeta_0 \varepsilon \text{Pe}}. \end{aligned}$$

The formula for the number $\text{Nu}|_{\zeta=\zeta_0}$ is obtained by replacing the quantity $\zeta_0 - \zeta$ by $1 - \eta$.

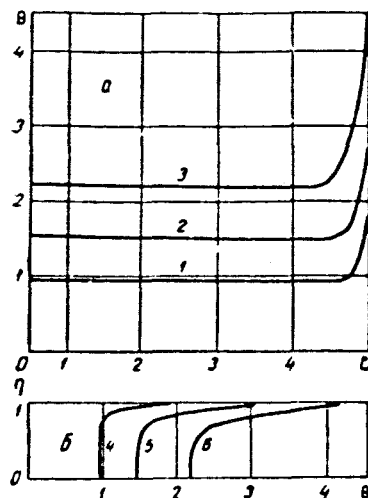


Figure 1

Temperature Distribution Along the Perimeter of a Rectangular Channel $z_0/h=5$ (α -along the wide side; δ -along the narrow side) For Uniform Temperature Field at the Entrance and for a Given Heat Current $Re=10^4$, $Pr=0.7$: 1.4- $\xi=20$; 2.5-50; 3.6-100.

Figure 1 presents the temperature distribution along the perimeter of a rectangular channel in different cross-sections, in the case of a uniform temperature field at the entrance. The cross-section of the channel is a rectangle with a side ratio of 1:5.

It can be seen from the graph that the temperature increases at the corners of the channel, while the width of the increased temperature region is the same in absolute magnitude, both for the short and for the long sides of the rectangle. Therefore, the greater the ratio of the rectangle sides (which represents the cross-section of the channel under consideration), the more extensive is the zone of increased temperature /427 along the entire short side.

Figure 2 shows the distribution of the Nu number along the short

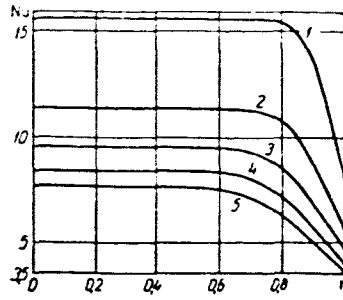


Figure 2

Nu Number Distribution Along the Perimeter (Along the Narrow Side) of a Rectangular Channel $z_0/h=5$ For a Uniform Temperature Field at the Entrance and for a Given Heat Current $Re=10^4$, $Pr=0.7$: 1- $\xi=10$, 2-20, 3-30, 4-40, 5-50.

side of a rectangular channel at different distances from the entrance. It can be seen from the figure that the Nu number decreases considerably at the corners.

The presence of a nonuniform temperature field at the channel entrance leads to a rather complex temperature distribution along the perimeter of a rectangular channel, particularly of a square channel (Figure 3). There is a much greater temperature increase at the corners than there is in the middle of the channel.

If the temperature field at the channel entrance is such that the temperature of layers closest to the wall is lower than the temperature of the liquid in the middle of the channel, then close to the entrance the mean mass temperature of the liquid is higher than the wall temperature. As one moves away from the entrance, at certain points in the given channel cross-section, the mean temperature of the liquid equals the wall temperature at this point, which corresponds to the number

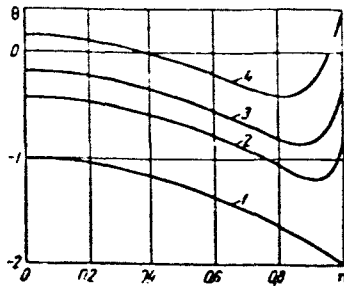


Figure 3

Temperature Distribution Along the Perimeter of a Square Channel for a Uniform Temperature Field at the Entrance, and for a Given Heat Current $Re=10^4$, $Pr=0.7$, $K_0=10$, $\gamma=-1$:
1- $\xi=0$, 2-5, 3-10, 4-20.

$Nu=\pm \infty$. Figure 4 presents the Nu number distribution along the sides of a square in the channel cross-sections located at a different distance from the entrance. It can be seen from an examination of this graph that for $\xi=5$ there is one break in the Nu number distribution, which corresponds to the point $\eta=0.45$; at a distance from the entrance of $\xi=10$ the number $Nu = \pm \infty$ at two points: $\eta=0.65$ and $\eta=0.97$.

At a given constant temperature of the wall, equation (4) must be solved under the following boundary and inlet conditions:

$$\left. \begin{array}{l} \eta = 1 \\ \eta = 0 \end{array} \right\} \theta = 0,$$

$$\xi = 0, \theta = \theta_w, \theta_z = (1 - \beta_w \gamma_i - \gamma_w \gamma_i^2)(1 - \beta_z \xi - \gamma_z \xi^2). \quad (11) \quad \underline{/428}$$

Here $\theta = (T_{cr} - T)/(T_{cr} - T_0)$.

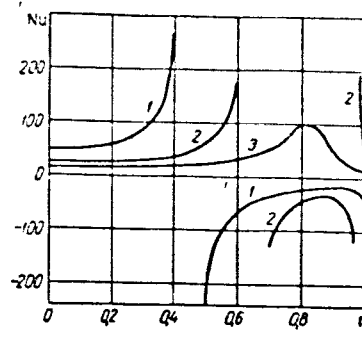


Figure 4

Nu Number Distribution Around the Perimeter of a Square Channel for a Uniform Temperature Field at the Entrance and for a Given Heat Current $Re=10^4$, $Pr=0.7$, $K_0=10$, $\gamma=-1$:
1- $\xi=5$, 2-10, 3-20.

The solution of equation (4) under inlet and boundary conditions (11) is the product of the solutions for the two following equations

$$\theta = \theta_y \theta_z:$$

$$\epsilon Pe \frac{\partial \theta_y}{\partial \xi} = \frac{\partial^2 \theta_y}{\partial \eta^2}, \quad \eta = \pm 1, \theta_y = 0, \quad \xi = 0, \theta_y = 1 - \beta_y \eta - \gamma_y \eta^2; \quad (12)$$

$$\epsilon Pe \frac{\partial \theta_z}{\partial \xi} = \frac{\partial^2 \theta_z}{\partial \zeta^2}, \quad \zeta = \pm 1, \theta_z = 0, \quad \xi = 0, \theta_z = 1 - \beta_z \zeta - \gamma_z \zeta^2. \quad (13)$$

Utilizing the solution of equation (12), obtained for the number $Re \gg 3 \cdot 10^3$ and $\xi \leq 100$, we obtain

$$\begin{aligned} \theta = & \left[\frac{8\gamma_y \xi}{\epsilon Pe} \right] \text{erfc} \left(\frac{1 - \eta}{2} \sqrt{\frac{\epsilon Pe}{\xi}} \right) + (\gamma_y + \beta_y - 1) \text{erfc} \left(\frac{1 - \eta}{2} \sqrt{\frac{\epsilon Pe}{\xi}} \right) + \\ & + 1 - \beta_y \eta - \gamma_y \eta^2 - \frac{2\gamma_z \xi}{\epsilon Pe} \left[\frac{8\gamma_z \xi}{\epsilon Pe} \right] \text{erfc} \left(\frac{1 - \zeta}{2} \sqrt{\frac{\epsilon Pe}{\xi}} \right) + (\gamma_z + \beta_z - 1) \text{erfc} \left(\frac{1 - \zeta}{2} \sqrt{\frac{\epsilon Pe}{\xi}} \right) + \\ & + 1 - \beta_z \zeta - \gamma_z \zeta^2 - \frac{2\gamma_z \xi}{\epsilon Pe}; \end{aligned} \quad (14)$$

$$\begin{aligned} \text{Nu}|_{r=1} &= \frac{A_1}{B}, \quad A_1 = \frac{\partial \theta}{\partial r} \Big|_{r=1}, \\ \text{Nu}|_{z=0} &= \frac{A_2}{B}, \quad A_2 = \frac{\partial \theta}{\partial z} \Big|_{z=0}, \end{aligned} \quad (15)$$

$$\begin{aligned} B &= \left[\frac{8}{3} \frac{\gamma_{z0}^2}{\pi (\pi \text{Pe})^2} + \frac{2(\gamma_{z0}^2 - 1)}{3} \sqrt{\frac{\xi}{\pi \text{Pe}}} + 1 - \frac{\gamma_{z0}^2}{3} - \right. \\ &\quad \left. - \frac{2\gamma_{z0}^2}{\pi \text{Pe}} \left[\frac{8}{3} \frac{\gamma_{\nu}}{\pi} \left(\frac{\xi}{\pi \text{Pe}} \right)^{1/2} + 2(\gamma_{\nu} - 1) \sqrt{\frac{\xi}{\pi \text{Pe}}} + 1 - \frac{\gamma_{\nu}}{3} - \frac{2\gamma_{\nu}^2}{\pi \text{Pe}} \right] \right]. \end{aligned}$$

Notation

x - longitudinal coordinate; y, z - transverse coordinates;
u - longitudinal velocity; v, w - transverse velocity components;
U - velocity of the liquid at the entrance cross-section of the channel; T - temperature of the liquid; a - temperature conductivity coefficient; * - thermal conductivity coefficient; 2h, 2z₀ - height and width of the rectangular channel; Re, Pe, Pr - Reynolds, Péclet, and Prandtl numbers.

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